MATHEMATICS METHODS Year 11
Section One:
Calculator-free

Your name ___Solutions___

Teacher name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using blue/black pen. Do not use erasable or gel pen.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1   (4 marks)

For the graph of $y = 10x - 2x^2 + 4$ determine the

(a)   coordinates of the y axis intercept.   (1 mark)

$$let \quad x = 0 \qquad (0, 4)$$

✓ correct
coordinate

(b)   equation of the line of symmetry   (2 marks)

$$L.O.S \qquad x = \frac{-b}{2a}$$

$$= \frac{-10}{2(-2)}$$

$$x = \frac{5}{2}$$

✓   ✓

{ ✓ states $x =$

✓ correct value

(c)   location and nature of the turning point.   (2 marks)

$$TP \quad \left(\frac{5}{2}, \frac{33}{2}\right) \quad ✓$$

states co-ordinate

Nature   maximum   ✓

substitution:
let $x = 2.5$

$$y = 10(2.5) - 2(2.5)^2 + 4$$

$$= \frac{33}{2}$$

See next page

The graph below contains two functions, $f(x) = \sqrt{5 + x}$ and $g(x)$.
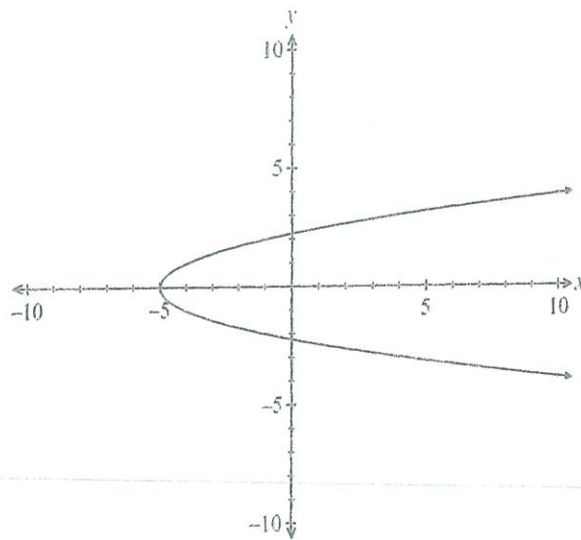
(a) Write the equation of $g(x)$ and explain why combining $f(x)$ and $g(x)$ as shown in the graph does not represent one single function. (2 marks)

$g(x) = -\sqrt{x + 5}$ ✓ Correct function.

• fails vertical line test   ✓ valid reason

• x values have more than one y value ⇒ not a function

(b) Write the domain of $f(x)$. (1 mark)

$\{x : x \in \mathbb{R} ; x \geqslant -5\}$ ✓ states domain correctly.

accept $x \geqslant -5$

(c) Write the range of $g(x)$. (1 mark)

$\{y : y \in \mathbb{R} ; y \leq 0\}$ ✓ states range correctly.

Note if (b) $\{x : x \in \mathbb{R} ; x > -5\}$, then $\{y : y \in \mathbb{R} ; y \leq 0\}$.

See next page

Question 3   (3 marks)

If $f(x) = x^3 + 4x^2 - 4x - 16 = (x - 2)(ax^2 + bx + c)$, then determine the values of a, b and c.

Linear
factors $(x-2)(x+2)(x+4)$ valid   method

Hence $(x-2)(x^2 + 6x + 8)$ ✓   Demonstrates understand-
                                                              ing of expansion

So $a = 1$
        $b = 6$ ✓   solves for a, b, c
        $c = 8$

NOTE:

Some candidates may use cubic factorisation then factorise
                                                              the quadratic.

$$\begin{array}{r} x^2 + 6x + 8 \\ (x-2) \overline{)\, x^3 + 4x^2 - 4x - 16} \\ -\quad x^3 - 2x^2 \\ \hline \end{array}$$ $(x-2)(x+2)(x+4)$

$6x^2 - 4x$

$-\quad 6x^2 - 12x$

$8x - 16$

$-\quad 8x - 16$

0

See next page

Question 4 (4 marks)

The three points $A(p + 11, 17)$, $B(-2, p)$ and $C(1, 7)$ are collinear. Determine the value(s) of p.

$$M_{AB} = \frac{p - 17}{-2 - (p+11)}$$

$$= \frac{p - 17}{-13 - p}$$

$$m_{BC} = \frac{7 - p}{1 + 2} \checkmark$$

$$= \frac{7 - p}{3}$$

Finds gradient of two sets of points

$$m_{AC} = \frac{10}{p + 10}$$

collinear : when $M_{AB} = m_{BC}$

$$\frac{p - 17}{-13 - p} = \frac{7 - p}{3}$$

$$\therefore \quad 3(p - 17) = (7 - p)(-13 - p) \checkmark \quad \text{Equates gradients}$$

$$3p - 51 = -91 - 7p + 13p + p^2$$

$$0 = -40 + 3p + p^2 \quad \checkmark \quad \text{Develops correct quadratic}$$

$$0 = (p + 8)(p - 5)$$

$$\therefore \quad p = -8 \quad \text{or} \quad 5 \quad \checkmark \quad \text{solves for both values. using Null Factor Law.}$$

End of questions

Christ Church Grammar School

MATHEMATICS METHODS Year 11
Section Two:
Calculator-assumed

Your name _____ Solutions _____

Teacher name _____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 25 minutes
Marks available: 24 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Question 5   **(4 marks)**

(a) The coordinates of the midpoint of $(3k - 1, 4 - 5k)$ and (x, y) are
$(4k - 1, 3.5 - 5k)$. Write an expression for x and y.   (2 marks)

midpoint : $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$\boxed{x}$.

$\boxed{4k-1} = \dfrac{\boxed{3k-1} + x}{2}$

$\boxed{8k - 2} = \boxed{3k-1 + x}$

$\boxed{x = 5k-1.}$ Final solution.

$\boxed{y}$ $\boxed{3.5 - 5k} = \dfrac{\boxed{4 - 5k + y}}{2}$

$\boxed{7 - 10k} = \boxed{4 - 5k + y}$

$\boxed{y = 3 - 5k.}$

✓ Demonstrates awareness of midpoint and applies

✓ Isolates x and y correctly.

(b) Show that the midpoint given in part (a) lies on the line with the equation
$5x + 4y = 9$ (2 marks)

let $x = (4k-1)$ $y = (3.5 - 5k)$

$5 (\boxed{4k-1}) + 4 (\boxed{3.5 - 5k}) = 9.$

$20k - 5 + 14 - 20k = 9.$

$9 = 9.$

Hence yes it does lie on the line.

✓ substitutes x and y co-ordinates correctly
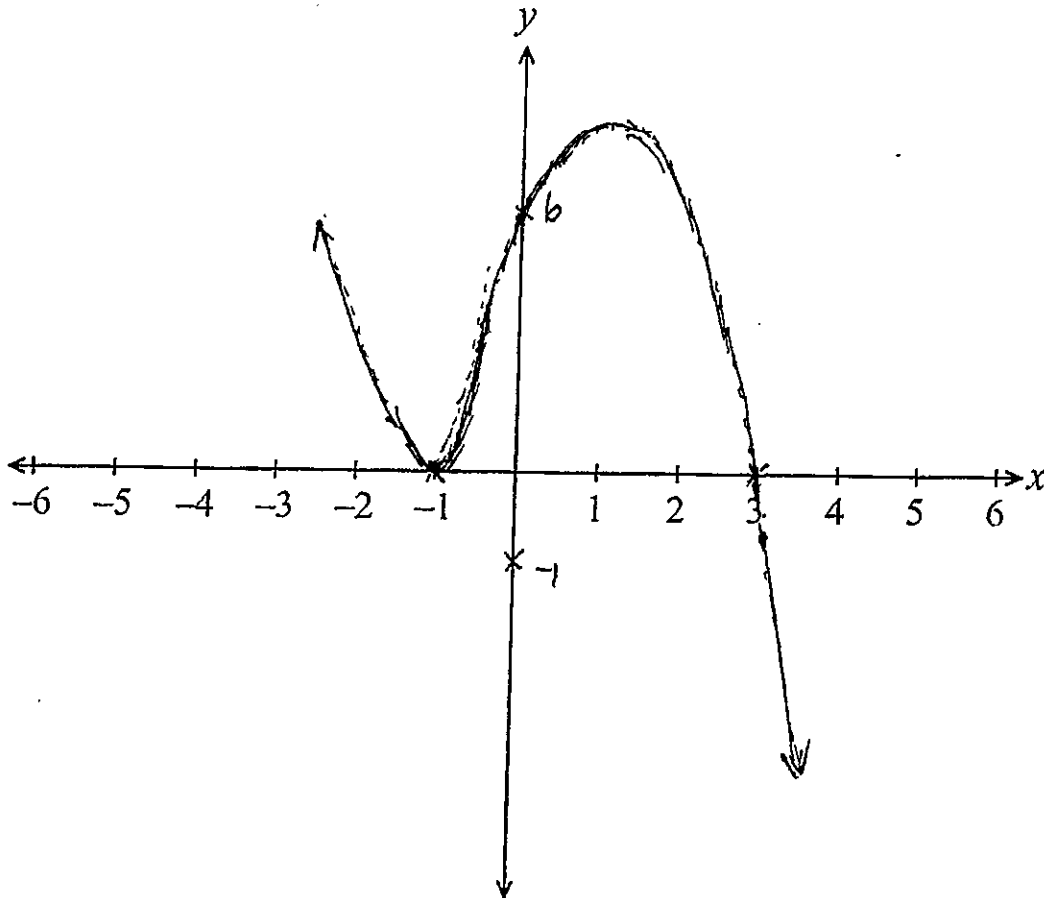
✓ completes algebraic statement correctly

$(9 = 9.)$

Question 6 (8 marks)

A function f is defined by $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. The graph of f has intercepts located at $(3,0)$, $(-1,0)$ and $(0,6)$ and a local minimum at $(-1,0)$.

(a) Sketch the function $y = f(x)$ on the axes below. (3 marks)

✓ Locates x intercepts @ $(-1,0)$ and $(3,0)$

✓ Correct shape — inverted cubic

  — repeated root @ $(-1,0)$

  —

✓ . y intercept at $(0,6)$ but not T.P.

See next page

Question 6 continued

(b)   Determine the values of a, b, c and d.   (4 marks)

$$f(x) = a(x-3)(x+1)^2$$

$$6 = a(0-3)(0+1)^2$$

$$6 = a(-3)(1)$$

Hence `a' = -2`.

$$f(x) = -2(x-3)(x+1)^2$$

$$= -2x^3 + 2x^2 + 10x + 6.$$

$$\begin{cases} a = -2 \\ b = 2 \\ c = 10 \end{cases}$$

$$d = 6$$

✓ Generates equation with `a` in linear factor form.

↙ solves for `a`

✓✓ correct values for all three b, c, d.

(c)   Comment on the behaviour of the function as $x \to \infty$.   (1 mark)

as $x \to \infty$, $y \to -\infty$. ✓

correct range.

Question 7 (5 marks)

Consider the diagram below, where M is the midpoint of OA and N is the midpoint of AB.
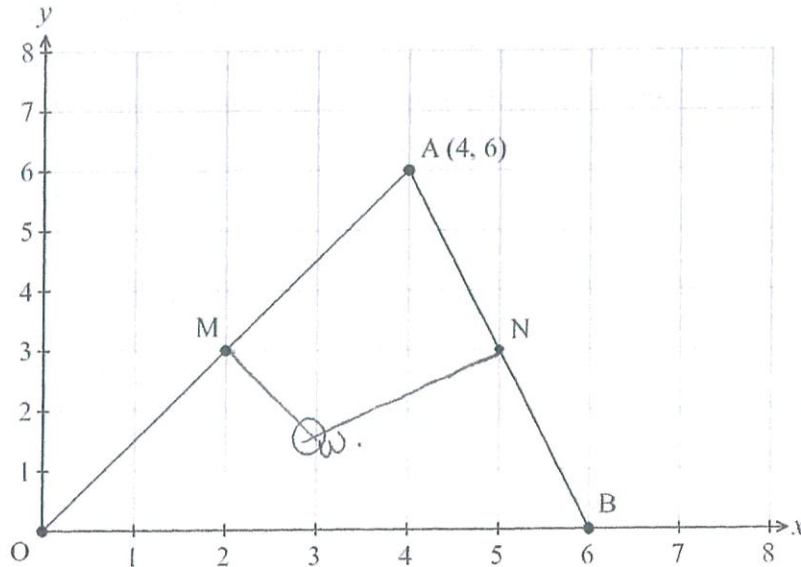
(a) Determine the gradient of the perpendicular bisector of OA. (1 mark)

$$m_{OA} = \tfrac{3}{2}$$

$$\therefore \perp \text{ bisector } m_{mw} = \boxed{-\tfrac{2}{3}}$$

✓ correct value for m

Let point W be the point where the perpendicular bisectors of OA and AB intersect.

(b) Determine the equation of the line MW. (2 marks)

$$m_{mw} = -\tfrac{2}{3}$$

$$\text{midpoint} = (2, 3)$$

$$3 = -\tfrac{2}{3}(2) + c$$

$$\therefore c = 4\tfrac{1}{3}$$

$$\boxed{y = -\tfrac{2}{3}x + 4\tfrac{1}{3}}$$

✓ solution

✓ method (valid)

(c) Determine the coordinates of point W. (2 marks)

line $x = 3$ ⊥ bisector of OB

$\Rightarrow W(3,)$

$y = -\tfrac{2}{3}(3) + 4\tfrac{1}{3}$

$y = -2 + 4\tfrac{1}{3}$

$= 2\tfrac{1}{3}$

✓ solution

$\boxed{(3, 2\tfrac{1}{3})}$ will

✓ valid method

·Variety of methods that may include

Point		Gradient	Equation
i) $(0,0)$ $(4,6)$		$OA = \tfrac{3}{2}$	$y = \tfrac{3}{2}x$
ii) $(4,6)(6,0)$		$AB = -3$	$y = -3x + 18$
iii ($2, 8$)		$mw = -\tfrac{2}{3}$	$y = -\tfrac{2}{3}x + 4\tfrac{1}{3}$
iv) $(5,3)$		$NW = \tfrac{1}{3}$	$y = \tfrac{1}{3}x + 1\tfrac{1}{3}$

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Question 8 (4 marks)

An archer's target is located 50 m away from her feet which are standing at (0,0) on a Cartesian plane. A circular target, of diameter 1 m, is 2.5 m off the ground, as shown below. She fires arrows at the target from a height of 1.5 m.
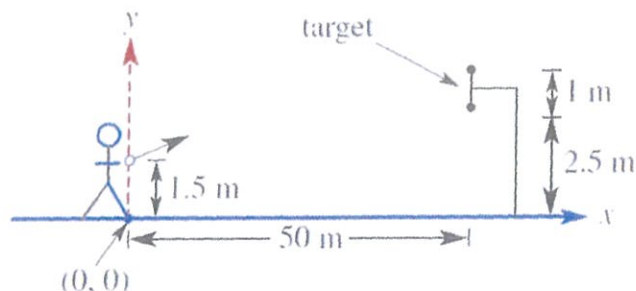
Assume that the distance travelled by the arrow can be represented using a linear function.

(a) Determine the gradient of the arrow's path to the bottom of the target. (1 mark)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{50} \quad \text{or} \quad 0.02$$

✓ Determines value.

(b) Determine the equation of the line for the arrow's path to the centre of the target. (2 marks)

$$c = 1.5 \qquad m = \frac{1.5}{50} = \frac{3}{100}$$

$$y = 0.03x + 1.5$$

✓ Gradient

✓ y intercept.

(c) Write an expression for the possible values of the gradient of the arrow's path for it to successfully hit the target. (1 mark)
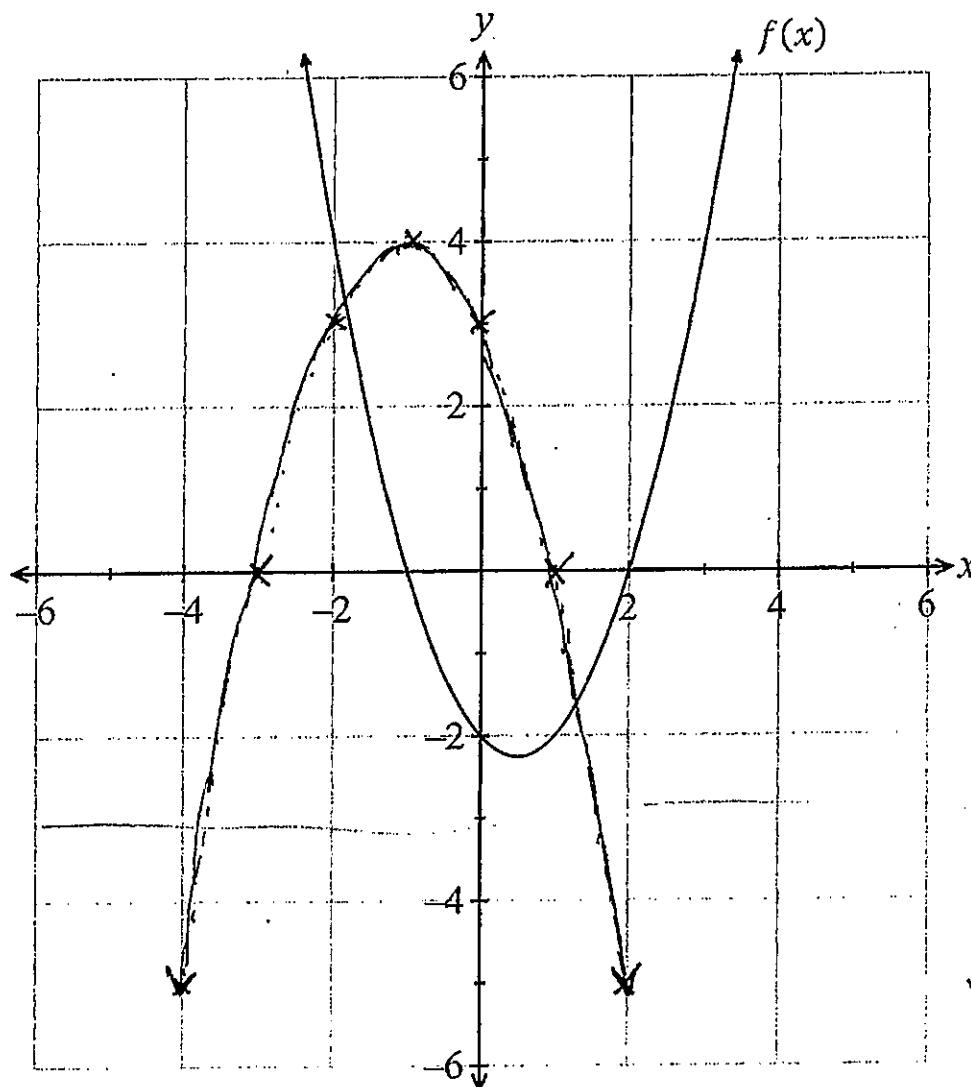
Archer needs to fire between

$$0.02 \le m \le 0.04$$

correct inequality

✓

See next page

Question 9 *3 (4 marks)*

The graph of $f(x) = (x+1)(x-2)$ is shown on the set of axes below.

✓ x ints

✓ correct shape and max TP·

✓ y intercept·

(a)   Draw the graph of $g(x) = -(x+1)^2 + 4$ on the above set of axes.   (3 marks)